TE 402 (Math) Lesson Study
Compare Fractions Lesson Plan
By Sandra Crespo with
her TE 402 students

Part I. Select your lesson math topic and clearly state your learning goals

Topic: Comparing Fractions with Unlike Denominators
Grade Level --- Grades 4-7
Mathematics Understanding Goals:
- Fractions tell us nothing about the size of the parts or the size of the whole; they tell us the relationship between the parts and the whole.
- Estimating fractions that are close to benchmark fractions – 0, 1, ½
- Fractions are equivalent when they represent the same amount or quantity
- Identifying, comparing, and ordering fractions

Common Core Standards:
Grades 3-5: Develop, Extend, and Apply Understanding of Fractions as Numbers
1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts.
2. Understand a fraction a/b as the quantity formed by a parts of size 1/b.
3. Understand Fractions as a number on a number line
4. Compare fractions by reasoning about their size.
Grade 3: NF3. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Grade 4: NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Grade 5: NF2. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

1. What are some important ideas underlying this topic?
-Making connections across strategies for comparing fractions while paying special attention to how each strategy relates to different meanings and representations of fractions (as a number in a number line, as a ratio, as a decimal, as a division, as part of a whole, …)

Fractions are tricky because fractions look very different from whole numbers. Fractions are written with a numerator and a denominator. So to start with, connecting fractions to anything that we know about whole numbers is important but often hard to do, because fractions seem to be very different from any other kinds of numbers students have studied by the time they are formally introduced to fractions. Additionally, there are different meanings and representations of fractions; the number five is five things and it is counted the same way no matter what the object is; but when we say one fifth, we can be saying quite a number of things – we could be saying one fifth of a pie (one piece out of five pieces), or one fifth in a number line; or 0.2 if we are thinking of decimals (or money – 20 cents to a dollar), or 20% if we're talking percents, and so on.

This lesson is about generating multiple strategies for comparing fractions. Fractions are easy to compare when they have the same denominators (for example, with 4/12 and 8/12 it is easy to see which one is bigger and which one is smaller). However, when we compare fractions with unlike denominators, things get a lot more complicated. When we compare 3/5 and 7/10 for example; how do we know which one is bigger, or whether the two are the exact same size? In this case using visual representations can help, and knowing how to make equivalent fractions also helps. In this latter case, we can see the connection between fifths and tenths, and realize that 3/5 can be re-written as 6/10 and so we now have made the problem into a comparison of fractions with the same denominators. Other fractions are a bit more challenging and so making sure that the two fractions have a common denominator is typically the way to go. However, this method, the same way as many other arithmetic procedures are often learned as steps to follow without much meaning.

2. **Why are these important** things for students to learn? (*Big ideas*)

- Different strategies for comparing fractions rely on different meanings and representations of fractions
- Exploring meanings and connections among procedures and representations are trademarks of a high-level learning task
-In real life people compare fractions in multiple ways and do not always rely on a single method
- Fractions are commonplace in our daily lives but also in the workplace
- Flexibility and Fluency with computational and problem-solving strategies are important in the study of mathematics – in this case, fluently means being able to use strategies when solving a particular example but being able to use it more generally to solve many more examples or adapt the strategy to use with similar kinds of examples; and flexibility means being able to recognize what strategy best fit the given situation, and adapt and combine methods as needed.

3. We might know several ways for comparing fractions, we can use drawings, percents, decimals, and of course the common denominator to compare fractions. However we tend to know these strategies independently and not always make a conscious effort to seek out connections among them. So thinking carefully about what are important connections among these strategies is important. In particular we should look more closely at the ways the less typical strategies are related to the more typical approach to comparing fractions —using common denominators.

So when we use the common denominator method – what is it that we're doing? In the case of comparing 2/3 and 3/5, what exactly are we doing when we use the common denominator method?

- First we assume that the two fractions belong to the same size whole; meaning that we're not talking about 2/3 of small pizza and 3/5 of a large pizza; our reference for these fractional parts are to the same size pizza – or whole, or unit.
- When the "whole" is not specified, the size of the whole is assumed to be 1
- So given these assumptions, then when we're comparing thirds and fifths, we know that 1/3 is definitely a bigger piece than 1/5 of the whole. If I'm comparing then 2/3 and 2/5 I can also tell that two thirds are more than two fifths. But the comparison between 2/3 and 3/5 is a bit more challenging.
- If we use our knowledge of equivalent fractions we can generate equivalent fractions to both 2/3 and 3/5 until we get equivalent fractions for both of these that share a common denominator
- So …
  2/3 = 4/6 = 6/9 = 8/12 = 10/15 = 12/18 = 14/21
  3/5 = 6/10 = 9/15 (Hey! I found a common denominator!) So I can compare 10/15 and 9/15 now; but also – I noticed that I can also compare 6/9 and 6/10 which are equivalent fractions to the 2/3 and 2/5 so … I can also see that ninths are bigger size pieces than tenths and that the same number of pieces in ninths and in tenths means that 6/9 is bigger than 6/10. So in the process of finding common denominators I also discovered equivalent fractions with a common numerator which means then we compare the sizes of the pieces as a way to compare fractions.

4. What might my students already understand about this topic? What sorts of informal and formal experiences might students have that can help them make sense of the main ideas?

Since we are trying this lesson with other TE 402 students, we can assume that they will be very familiar with the common denominator method. Because they studied this quite some time ago, it is possible that some of them will use the cross multiply step to make the two fractions have a common denominator and may not realize that this connects to the equivalent fractions idea. So in the case of 2/3 and 3/5; basically we multiply the denominators 5x3 = 15 and then cross multiply 2x5 and 3x3 to make the numerators of the equivalent fractions -→ we end up with 10/15 and 9/15. These steps conceal the idea that what we're really doing is finding equivalent fractions for both fractions until we get a common denominator that will allow us to compare. Using these step is a quick and easy way of converting fractions to their equivalents, but if we have more than one fraction to compare … not sure that this is the best approach to use. Say that we have to compare and order the following fractions 2/3, 3/5, 5/7, 7/10 and 8/11, would this cross multiply procedure still work?

So our guess is that the common denominator method will be the most dominant method, but there are different ways to carry out this method, so one thing we can be curious about is how many different methods are there for the common denominator strategy?

We can also expect visual representations, with the fractions we are using in this task we might see the rectangular drawing (brownies) more so than the circular representation (pizza or pie) one. Not sure about the discrete models of fractions (M&M's or little cookies) whether these will be used. When we did this in our class, no groups used discrete representations, so it will be interesting to see if the other 402 class will use those as visual models. We should also expect that TE 402 students would be able to translate fractions to percents and decimals, and then use those to more easily compare the fractions.

The part that will probably be more challenging is not generating multiple methods, but the part about seeing connections among the methods. We might want to push for connections by asking them to consider connections in the representations, as well as meanings of fractions (i.e., the size of the "whole" and the "parts" across methods, and also what is the meaning of comparing fractions across the methods).

Part II. Select a High Level math task and Work on it Yourself and with Others

Task:

Materials:

- graph paper
- poster paper *from your teacher*

Task:

As a group, use four different strategies to compare $\frac{6}{9}$ and $\frac{5}{8}$ Make many connections across strategies

Final Product:

Your group should make a poster that:

1. **Shows each strategy and how you used the strategy to compare the fractions. You can use grids, pictures, charts, written explanations or any other tools you need to make your explanation clear and easy to understand.**
2. Shows connections across strategies. Use color, arrows, and other math tools to communicate your group's mathematical reasoning.
3. Shows everyone's ideas and is well organized.

FOUR METHODS for comparing fractions (This is our collective work from last class)

1. Comparing Fractions via the Common Denominator method
   Making the two fractions have the same denominator via multiplying/dividing; using common denominator of **24**; the comparison becomes that of comparing 16/24 and 15/24; with 72 as the common denominator, the equivalent fractions become: 48/72 and 45/72

2. Comparing the "unshaded" parts – or the two other fractional parts that complete the whole. Instead of comparing 6/9 and 5/8; compare 3/9 and 3/8, which then creates the situation of comparing fractions with equal numerators. So in this case we compare the size of the parts.

3. Converting fractions to decimals or percents and comparing them that way

4. Using a visual representation – a rectangular array or circular visual, or number line – but those require that we already know which one is smaller/bigger, in order to accurately represent the comparison.

5. Decomposing the fractions into unit fractions and using those to compare;
   for example: 6/9 = 1/9+1/9+1/9+1/9+1/9+1/9 = 5/9 + 1/9 and compare this to 5/8 - ; or some other combinations.

Part III. LESSON SCRIPT

Lesson Plan Template

Sample Outline for a Daily Lesson Plan

Date: *March 22/2012*

Overall lesson topic/title and purpose *(What do I want students to learn?)*

- Comparing fractions with unlike denominators
- Purpose of the lesson is to bring together a collection of concepts and ideas related to fractions that tend to be studied discretely across elementary grades. The goal is to contrast methods for comparing fractions and see how these methods are similar. In this lessons students will:
- Learn some of the processes that make groupwork productive, such as establishing norms and expectations for the group work and designing a high level task that is groupworthy – that everyone in the group can contribute to and learn from.
- Add a few more strategies to their repertoire of strategies for comparing fractions
- Be able to explain how each strategy ensures that the two fractions are referencing the same size whole and/or the same size pieces.
- Be able to explain the meaning of fraction that can be associated with each method of comparison

Rationale *(Why is it worthwhile? How does it link to Standards, Benchmarks, GLCE, Curriculum Guidelines?)*

GLCE – Strand 1: Number & Operations

Meaning, notation, place value and comparisons (ME), Number relationships and meaning of operations (MR), Fluency with operations and estimations (FL).

- *N.ME.05 Order rational numbers and place them in a number line*
- *N.ME. 06.06 Represent rational numbers as fractions or terminating decimals when possible and translate between these representations.*

Common Core Standards:

Grade 4: NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Grade 5: NF2. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Goals/Objectives for today's lesson:

- math goal: students generate multiple methods for comparing fractions and make connections between methods.

- social goal: students work together productively, share their thinking and check each other's understandings

Materials & supplies needed:

- *For each group: task card, graph paper, large poster board*
- *Other materials available – sticky notes, glue sticks, tiles, strips, color markers*

Procedures and approximate time allocated for each event	***Academic, Social and Linguistic Support during each event*** *(Lauch/Explore/Discuss)*
Launch *(8 minutes)*	
We will start the lesson by setting the group norms and expectations for today's lesson. In particular the norm that each group needs to work together and make sure to check with each other's understandings. Any group member should be able to represent the group's insights about the task. Go over role assignment and responsibilities for each role.	*Monitor groups' understanding of the task; the word "strategy" might need to be reworded for some students.* *The symbols <, >, and = might help clarify*

The goal of the lesson will be introduced by using a series of powerpoint slides reminding students that fractions have different meanings, and that they can be represented in various ways. Similarly we can compare fractions in various ways as well, so announce that today we will be doing a task using what we know about fractions, about ordering and comparing fractions, to extend our ideas about methods for comparing fractions and relationships we can make among these methods.

The comparing fractions task will be projected and teacher announces:
"Now, here is the task you will work on as a group. You have a print out of this task in each of the group's folders, the resource monitors for each of the groups need to come get your folders. The first thing the groups need to make sure to do is to read carefully the task and make sure you all have a clear understanding of what the task is requesting. Remember that you can always call me for help, but need to make sure that the resource manager does that, but also that I'm expecting that you are calling me for group questions"

"Time wise, you have 20 minutes for group work, I will give you a time warning at the 15 mins mark so that you can make sure you wrap up your work."

Explore *(20 minutes)*

Phase 1: Understanding the task --- I'm expecting to see the groups trying to get clear on the task and sharing with each other their ideas. The groups will organize themselves differently, some will stay together and work out a strategy at a time, while others will do the task separately and then share with each other.
Phase 2: Working on the task --- Most groups will begin the task working with diagrams but quickly realize that the diagrams are not helpful with comparing these fractions and so they will switch to symbolic manipulation. They will likely approach these computations very procedurally, until someone raises questions about what they are doing and how things make sense.
Phase 3: Preparing the poster --- The groups will have to organize their work so that it can be shared broadly. The sequencing of the strategies in the poster will likely follow the sequence in which the group came up with them. I'm expecting that most groups will do: 1. Denominator strategy, 2. Percents or decimals, 3. Number line, 4. Fractional part drawing (rectangle)

I imagine that I will have to call a huddle in order to remind the groups that the task requests that they be explicit about connections between methods. I will specifically request that they consider what method relates to which meaning of fraction, and what it is their method does to the "pieces" and to the "whole".

Discuss *(12 minutes)*

Once posters are ready and posted around the room; I will call on one of the roles to represent some of the group's thinking about the work they did to compare fractions.

The main point for discussion is to highlight the goals for our lessons-

the task.

To ensure students remain engaged in the task, we could call a "huddle" of one of the group members to remind them that the task is not to only generate strategies for comparing fractions, but they need to show and explain connections they see among those strategies.

Several reminders about the group norms and expectations.

If groups are having difficulty with the given fractions, I will ask them to work with a simpler set of fractions at first to get their heads around the task; how might they compare 2/3 and 3/5; how do we know which one is bigger, or if they are the same size?
….

As an extended challenge, students can be asked to compare rational numbers that are written as fractions and as decimals. For example, compare 0.65 and 6/7. Which of their methods will they use to compare these ones? Is there a particular method that they would not use? Why?

I will announce that there are common strategies across all of the groups, can we identify which strategies were used by all of the groups. So we will first work with the strategies all of the groups used and make connections across those. I will call on students using their roles – group 2 (facilitator) please tell us about connections your group saw between the common denominator method and the converting fractions to decimals method? What meaning of fraction is behind each of these methods? (Fractions as parts of a whole? Fractions as quotients, fractions as measures, fractions as ratio …). ***How will I summarize the main ideas of the lesson?*** *Today we cover a huge territory of ideas related to fractions and ways to compare them, let's spend these last few minutes to reflect on what we have learned today in relation to our lesson's goal. What new insights did you gain in this lesson about:* - *doing math as a group* - *comparing fractions* ***<u>Transition to next learning activity</u>***	
Assessment *(Include comments about how you are going to use information from this class through formative assessment and collected student work samples to inform future lessons.)* The group's final product will be a poster. This poster will provide evidence of the group's work. The poster will be evaluated for completion (4 strategies are included and connections among them are visible) and also for the quality of the work and especially the explanations that are offered about the kinds of connections the group was able to make across strategies. And, how are these visible in the poster. Assessment will also be ongoing via observations. I will be observing the group's workings for students who participate and do not participate across the three different phases detailed in the 'explore' section of the plan. Whose ideas are taken up and whose ideas are not taken up by the group will give me a sense about possible issues of status among the group members.	Academic, Social, and Linguistic Support during assessment

Part IV. Write a CLASSROOM DIALOGUE of your envisioned class discussion
After completing your lesson script, zoom in to the moment when you bring the class together to share and discuss the mathematics work the students have done on the task. Write a classroom dialogue (in the form of a transcript) to illustrate the *selecting*, *sequencing*, and *connecting* of students' mathematical ideas for the mathematics task in your lesson. Construct a dialogue that helps illustrate what you are expecting will be hard to do when you go to debrief students' thinking on your task that you will need to address so that the discussion accomplishes your lesson's mathematical learning goals.

This exercise is about imagining what teachers do and say in relation to what students say and do. What happens when teachers ask beautifully constructed questions … sometimes they have to repeat them, sometimes they have to rephrase, and so on. This part of the assignment is about creating a dialogue representation of the discussion section of your lesson plan with **at least 10 Speaking turns for students (Keep in mind that students can add on to each other's ideas! A turn does not have to be counted as an interaction between a student and a teacher)**. Make sure that you include your three focal students (from interview project) in this dialogue so as to represent what you know so far about them as math students.

** Note that you will have opportunities during class to work with these kinds of lesson dialogues – some of these you can find in our books already – but in some mathematics textbooks they include these kinds of dialogues in the teacher guides such as the K-5 textbook series "Investigations" and "Connected Mathematics."

Part V. Reflections
In this part write a reflection on your lesson planning. What aspects of it were easy and challenging? How prepared do you think you are to teach this lesson, should you have an opportunity to do so? What surprised you? What part of the lesson are you still unsure and puzzling over? How might you follow up this lesson (and why)?

If you were able to teach this lesson to a group of students in your field placement, these are some questions you might also report on in your reflections:

- What went as you expected and what didn't? Did you stick to your lesson plan or did you make changes on the fly (describe and explain)?
- What do you think your students learned during your lesson (and what evidence do you have)?
- What insights did you gain about students, mathematics, teaching mathematics, and yourself as a teacher from planning and also teaching this lesson?
- What questions do you now have after this teaching experience?